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DETERMINATION OF A CONIC FROM GIVEN CONDITIONS.

By Dr. J. Harrington Boyd, Chicago, Ill.

In this paper is given a method for the determination of a conic when the coordinates of one of its foci, the abscissa of the other, and the condition that the conic touches two given parallel lines are given.*

Take the given focus as the origin, then, if the center of the conic lie on the x-axis the equation to the conic in rectangular coordinates will be

$$x^2 + y^2 = e^2(\sigma + x)^2 \tag{1}$$

where e is the eccentricity and σ the distance from the origin to the directrix.

The equation of the conic, when its axes are revolved through an arbitrary angle λ, may be found by putting for x and y, respectively,

$$x \cos\lambda - y \sin\lambda \text{ and } x \sin\lambda + y \cos\lambda ; \tag{2}$$

whence

$$x^2 + y^2 = e^2(\sigma + x\cos\lambda - y\sin\lambda)^2 . \tag{3}$$

The major axis of the conic will now pass through the origin making an angle λ with the x-axis.

Represent the two given lines which are parallel to the y-axis by the equations

$$x = -l \text{ and } x = +l' . \tag{4}$$

If the conic represented by (3) touches the line $x = -l$, then the following quadratic equation must have equal roots :—

$$(1 - e^2\sin^2\lambda)\, y^2 + 2e^2(\sigma - l\cos\lambda)\sin\lambda \,.\, y + l^2 - e^2\sigma^2 - e^2l^2\cos^2\lambda$$
$$+ 2e^2 l\sigma\cos\lambda = 0 . \tag{5}$$

The condition that (5) have equal roots is

$$e^4(\sigma - l\cos\lambda)^2\sin^2\lambda = (1 - e^2\sin^2\lambda)(l^2 - e^2\sigma^2 - e^2l^2\cos^2\lambda + 2e^2 l\sigma\cos\lambda)$$

or

$$l^2 = e^2(\sigma^2 + l^2 - 2\sigma l\cos\lambda) . \tag{6}$$

Similarly the condition that the conic in (3) touches the line $x = +l'$ will be

$$l'^2 = e^2(\sigma^2 + l'^2 + 2\sigma l'\cos\lambda) . \tag{7}$$

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* This problem is interesting on account of its applicability in the determination of orbits of double stars, and was suggested, through Dr. See, by Prof. Burnham of the Yerkes Observatory.

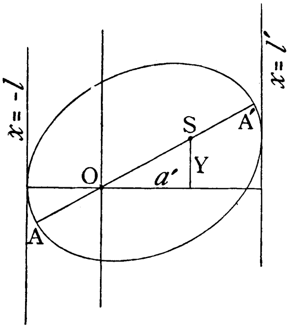
The third equation needed to determine the unknown quantities e, l, and λ is found in the following manner :—

The equation to the major axis of the conic may be written in the form

$$\frac{x}{\cos\lambda} = \frac{y}{\sin\lambda} = \rho\,, \tag{8}$$

where ρ is the distance of the point (x, y) from the origin O.

Substituting the values of x and y from (8) in (3) we get the following equation in ρ, whose roots are the distances AO and $A'O$ (see figure).



Here

$$\pm\,\rho = e\sigma + e\rho\cos 2\lambda\,, \tag{9}$$

whence

$$OA = \frac{e\sigma}{1 - e\cos 2\lambda} \quad\text{and}\quad -\,OA' = \frac{e\sigma}{1 + e\cos 2\lambda}\,,$$

$$AA' = \frac{2e\sigma}{1 - e^2\cos^2 2\lambda}\,,\qquad OS = \frac{2e^2\sigma}{1 - e^2\cos^2 2\lambda}\,. \tag{10}$$

Consider now the coordinates of the focus S to be (a', Y), then

$$Y = OS\sin\lambda = OM\tan\lambda\,,$$

or

$$\frac{2e^2\sigma}{1 - e^2\cos^2 2\lambda} = \frac{a'}{\cos\lambda}\,. \tag{11}$$

We have now equations (6), (7), and (11) to solve for e, σ, and λ. Eliminating e^2 first from (6) and (7), then from (6) and (11) we get the following equations :

$$(l' - l)\,\sigma = 2ll'\cos\lambda\,,\qquad \cos\lambda = a\sigma\,,\qquad a = \frac{l' - l}{2ll'}\,,$$

$$2l\,(a' + l)\cos\lambda\sigma + l^2a'\cos^2 2\lambda - a'\sigma^2 - a'l^2 = 0\,. \tag{12}$$

Eliminating $\cos\lambda$ from the equations in (12),

$$\sigma = \pm \frac{1}{2la^2}\sqrt{1 - 2la\left[1 + \frac{l}{a'}\right] + 4a^2l^2}\,; \tag{13}$$

and, therefore,

$$\cos\lambda = \pm \frac{1}{2la}\sqrt{1 - 2la\left[1 + \frac{l}{a'}\right] + 4a^2l^2}\,.$$

The last equation in (13) enables one to draw the major axis of the conic. It follows from equation (11) that the expression for the eccentricity is

$$e^2 = \frac{a'}{2\sigma\cos\lambda + a'\cos^2 2\lambda}\,. \tag{14}$$

So, when $\cos\lambda$ and σ have been calculated by means of (13), then e can be found from (14) and the major axis from

$$AA' = \frac{2e\sigma}{1 - e^2\cos^2 2\lambda}\,.$$

The two signs in (13) correspond to the two positions which the conic may occupy with respect to the x-axis, namely, above and below it.

University of Chicago, *February* 20, 1895.